Reflex

From acquisition to automaticity:
The Reflex solution for math fact mastery

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Introduction: math fact fluency

"Use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing…the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts…"

National Math Panel Final Report (2008), Principal Messages

Math fact fluency refers to the ability to recall the basic facts in all four operations accurately, quickly and effortlessly. When students achieve *automaticity* with these facts, they have attained a level of mastery that enables them to retrieve them from long-term memory without conscious effort or attention. Brain imaging studies have revealed how the progression from effortful processes such as finger counting and explicit strategy use to automatized retrieval is associated with actual changes in the regions of the brain involved in mathematical computation (e.g., Rivera, Reiss, Eckert, & Menon, 2005). Through automaticity, students free up their working memory and can devote it to problem solving and learning new concepts and skills (Geary, 1994).

Extensive research has demonstrated the critical role of fact fluency in elementary school level mathematics and beyond (e.g., Isaacs & Carroll, 1999; Kail & Hall, 1997; Miller & Heyward, 1992; Royer et al, 1999; Woodward, 2006; Zentall 1990). In this research, mental chronometry – the precise measurement of the speed with which a student can recall a given fact -- is the typical method used to evaluate fluency. The importance of retrieval speed as a measure of fluency is underscored by studies that show it is a significant predictor of performance on standardized tests, including tests such as the SAT where calculator usage is permitted (Royer et al., 1999). Furthermore, the significance of fact retrieval speed as a predictor of performance is not limited to test items that directly assess computation – it predicts performance on math concept problems, word problems, data interpretation problems, and mathematical reasoning items as well.

Research over the past decade has also shown, however, that many children in the United States never achieve sufficient proficiency with math facts, and those who do typically achieve it later than their peers in nations with higher mathematics achievement (Gersten et al, 2009; National Mathematics Advisory Panel, 2008). To address this issue, recent national curriculum standards and guidelines have highlighted automaticity with math facts as a core objective of elementary mathematics education, including:

- NCTM's Curriculum Focal Points (2006)
- National Math Advisory Panel's Core Principles of Math Instruction (2008)
- Common Core Standards for Mathematics (2010)

It is important to bear in mind that these organizations are not advocating automaticity as a substitute or replacement for conceptual understanding in mathematics. Rather, conceptual understanding and fact fluency are mutually supportive, and should not be seen as competing for class time (National Mathematics Advisory Panel, 2008).

Provided an efficient and effective approach to fluency development is used, students can master their math facts in all four operations through a series of short practice sessions. By devoting a small part of each classroom session to fluency development, educators can preserve the majority of time for developing conceptual understanding on a solid foundation of fact mastery.

Successful fact fluency development can also be very important on another level. It provides an excellent opportunity for students to realize the importance of effort in their academic success - as opposed to attributing success to some form of innate talent or ability - and adopt a more positive attitude towards mathematics and their own competency with it. Conversely, when students do invest significant effort in developing fluency but are hampered by practice methods that have limited effectiveness, the opposite effect can occur and the "success is due to talent" mindset may unfortunately be reinforced.

To understand why, we'll take a look at what research shows limited fact fluency might feel like for a hypothetical student, "Maria".

Maria is having some challenges at school, particularly in math.

Although she can successfully give answers to many basic addition and subtraction problems given time, she frequently has to use her fingers to do so. Overall, her math fact fluency is limited and inconsistent at times even with those facts she appears to know. This seems to be affecting her proficiency with many of the mathematical procedures being taught in her class.

For example, although she understands how to do addition and subtraction problems with multiple digits, she has repeated trouble with regrouping and often makes 'careless' errors. She is starting to show signs of difficulty progressing to more advanced concepts and skills.

Her teacher reports she is easily distracted.

Last month, Maria's teacher started using timed worksheets with all her students to develop their fact fluency. Some students made gains in the total number of problems they completed per minute, but it's mostly the children who already seemed to be fairly fluent with their facts to begin with. (Research shows that timed practice is only effective for the facts that students can already successfully recall from memory (Hasselbring, Goin, & Bransford, 1987). Some increase in retrieval speed may occur over time **if** the performance benchmarks are well matched to that student's current fluency. If the performance benchmarks are too easy for a given student, for example, they can actually cause speed to **decrease** (Haughton, 1972)).

None of this seems to be making much of a difference for Maria and the other students in the class who are struggling with math facts, however. Timed worksheets aren't an effective means for student to learn new facts or transition away from effortful strategies such as finger counting (Miller & Heyward, 1992).

Maria's teacher has taught her class some strategies for using facts they know to derive answers to those they don't. These strategies have helped Maria, but she finds that to use them successfully she often has to pay careful attention while she's figuring out each fact (Woodward, 2006). This slows her down when she's learning something new or doing homework with a procedure that requires many math facts. Her limited existing fluency also leaves her with a fairly small pool of "known" facts to use the strategies with. As a result, she still frequently tends to fall back to counting on her fingers or making tally marks in page margins.

At this point, Maria's difficulties may prompt a more concentrated intervention. Daily sessions of individual flashcard-based work may begin. Perhaps her parents participate as well, with additional extended flashcard drill sessions taking place at home. Maria is encouraged to "try harder" and "be more careful" to reduce her "careless" errors.

The problem here is not with Maria, however -- it is that effective fluency development requires:

- **Learning new math facts in a way that facilitates progress to memory-based retrieval**

  Effective learning of new facts requires systematic introduction of a limited number of these facts at a time, using appropriate strategies for that student and exercises that catalyze fluid memory-based retrieval and inhibit the student from falling back to effortful methods such as finger counting.

- **Strengthening the association of the fact in memory to make retrieval automatic**

  Efficient practice requires continuous adjustment of the difficulty of the retrieval process, to keep it just within the student's current ability.

This is impractical to do with flashcards, and so typically the student is simply drilled over and over on a large set of facts in a repetitive fashion. As with timed worksheets, significant gains often fail to result for struggling students, regardless of the effort and time spent here.

How does all this feel for Maria? In the classroom, more and more of the concepts and skills she will be expected to learn will involve a significant number of math facts (Royer et al, 1999). For her, this means effort and careful attention at every step and substep of a new process, just to get the math fact parts correct. She finds it hard to follow and participate in classroom discussions.

She has also begun to notice that she seems to be slower than other kids at math, for whom math facts seem to come very easily. "I'm just not good at math", she decides. She starts trying to go through her math homework quickly, so that it is over as soon as possible (Ashcraft, Krause & Hopko, 2007) or simply fails to do it because she perceives that it is too difficult to complete successfully (Skinner, Pappas, & Davis, 2005). She may even reduce her visible use of processes like finger counting, but this doesn't mean that she is developing fluency – rather, she simply guesses more frequently (Geary et al, 2007).

Maria is starting to give up on math.

And it's not that she hasn't tried, or that her teacher and parents haven't tried. The unfortunate consequence of all the ineffective effort and intervention is that it **further** convinces her that she "isn't good at math" -- that math must be the result of talent, not effort. This has been termed the "labor in vain" effect (Nelson & Leonesio, 1988). "How can success in math be the result of effort", she thinks, "when I've tried so hard and it feels like I'm going backwards?"

How do we solve a problem like Maria's?

Math fact fluency poses a significant challenge to educators – on the one hand, research shows that it plays a critical role in students' success in elementary mathematics and beyond. For example, differences in addition fact retrieval measured as early as Grade 1 have been shown to predict membership in high versus low achieving groups in subsequent years (Geary, 2009). On the other, research also reveals the limitations of traditional fluency development methods such as flashcards and worksheets to satisfactorily address the fluency deficit impeding struggling students.

Yet, in this challenge lies a perfect opportunity to convincingly demonstrate to students how effort **can** lead to success in mathematics. By virtue of its nature, fact fluency is one of the rare domains where measurable improvement can occur in almost every session, provided an effective, efficient system is employed. These steady gains can be an incredibly powerful motivator for the student, instilling confidence that goes beyond the realm of math facts and builds perseverance for tackling advanced, challenging schoolwork in all subjects.

The critical element here is the effectiveness and efficiency of the methods used to develop students' fact mastery.

Educational technology can play an important role in disseminating powerful, research-validated instructional techniques by making them practical to implement in the classroom on a regular basis. In this regard, the National Math Panel's Final Report recommended the use of computer-based software for developing math fact automaticity, provided that the software is well designed.
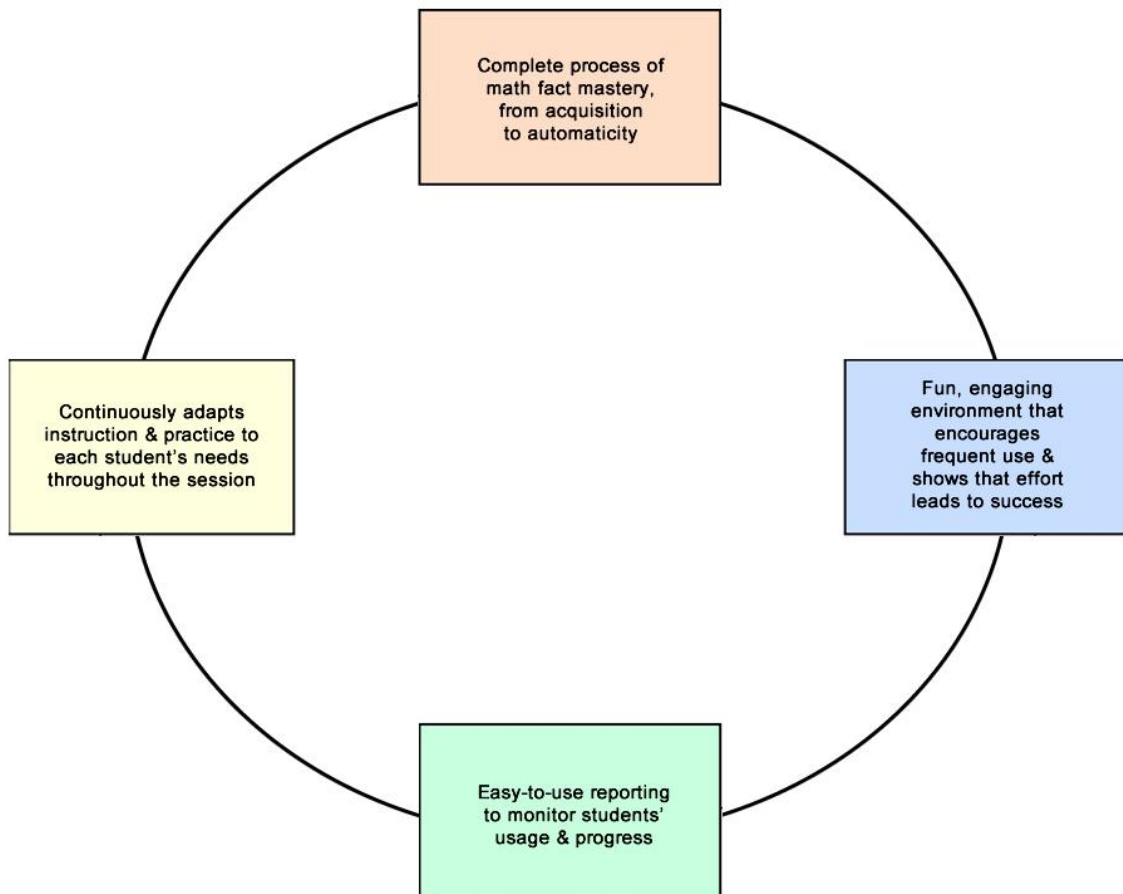

This white paper describes how the Reflex system uses research-proven methods and innovative technology to provide an effective and efficient solution for developing students' math fact fluency to full automaticity.

Reflex: the complete solution for fluency development

ExploreLearning® Reflex® is a next generation math fact fluency development system that:

1. Covers the complete process of fact mastery, from initial acquisition of previously unknown facts through to automaticity.

2. Continuously differentiates instruction and adapts practice to each student's current ability and needs, throughout the entire session.

3. Generates a fun, motivational environment for students, one that encourages frequent use and reinforces the connection between effort and success in mathematics.

4. Provides educators with intuitive, insightful reports to monitor fluency gains and system usage.

Complete process of math fact mastery, from acquisition to automaticity

Continuously adapts instruction & practice to each student's needs throughout the session

Fun, engaging environment that encourages frequent use & shows that effort leads to success

Easy-to-use reporting to monitor students' usage & progress

1. Math fact mastery: from acquisition to automaticity

Reflex covers the complete process of math fact mastery, including:

a) systematic introduction of small sets of new facts using appropriate strategies;

b) development of the student's preliminary ability to recall these new facts from memory;

c) progression to timed retrieval once the student has demonstrated readiness;

d) automatization through game-based practice, wherein facts are recalled while the student's working memory is increasingly loaded with game-based tasks

Complete process of
math fact mastery,
from acquisition
to automaticity

This section of the white paper describes the main stages used by Reflex to develop fluency with a given math fact when a student has little or no pre-existing ability to recall it.

A. Systematic introduction of small sets of new facts using appropriate strategies

Reflex uses a variety of explicit strategies to teach new facts, including the commutative property, fact families and rule-based patterns (e.g., multiplication by ten).

During their initial usage of Reflex, students receive introductory lessons on these strategies.
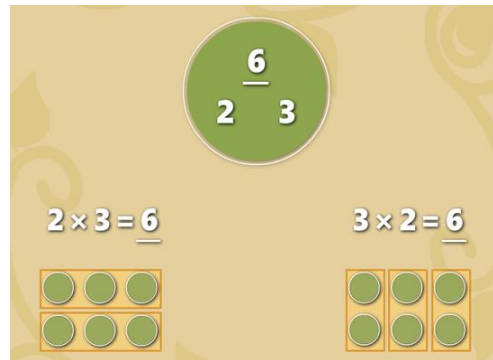
The system then uses these strategies to build on the student's existing fluency in an explicit manner. For example, a student may have some fluency with 3 x 6, but be inconsistent with 6 x 3. Reflex will leverage this existing ability and support the student in making the connection between the known and unknown facts using the commutative property.

In future sessions, Reflex extends this approach to fact families. The system will identify a candidate family that the student has some pre-existing ability with. For example, prior work in Reflex may have developed the student's fluency with the addition facts 5 + 8 and 8 + 5. Reflex now continues building on the student's knowledge by connecting it to the related subtraction facts 13 – 5 and 13 – 8.

In all cases, the most appropriate strategy for that student is selected by Reflex based on the type of fact and the student's demonstrated ability with it and with related facts.

Once students are ready to begin using a given strategy, they will progress to the first stage of acquisition practice. Here, they must recall both the known and new facts from memory and type them in:


Reflex's approach here is based on the *Cover, Copy and Compare* procedure. Numerous studies have validated the effectiveness of this approach for initial fact acquisition for students with and without learning disabilities (Menon, 2010). In addition, recalling the set of facts as a group helps prevent the possibility that students are only "juggling" these facts in short term memory or memorizing the facts solely as discrete entities that are unconnected to one another.

The use of fact families is also critical in terms of the system's ability to efficiently cover all four operations. Students' fluency with subtraction and division typically lags behind addition and multiplication (e.g., Hatfield, Edwards & Bitter, 1997; Menon, 2010), and this impacts their proficiency and comfort with topics such as fractions (Gersten et al., 2009). By simultaneously addressing subtraction with addition, and division with multiplication, Reflex is able to support students in mastering the basic arithmetic facts across all four operations in an expedient manner.

B. Progressively developing and strengthening the student's ability to recall new facts

Once the student has demonstrated a preliminary ability to recall the new facts under study via the selected strategy, Reflex transitions to the next phase of fluency development.

Here, the student completes different forms of "open sentences" (Groen & Poll, 1973; Weaver, 1973). The first type of open sentence has two 'blanks' that must be completed using the facts under study. For example, if one of the facts the student was studying was 6 x 7 = 42, the first open sentence might be 6 x __ = ___. The use of two open fields hinders the student from regressing to non-memory-based strategies, such as finger counting.
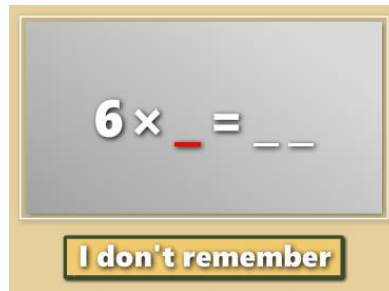
The second type of open sentence uses a single blank, such as __ x 7 = 42. The open field may appear in different places, eventually concluding with 6 x 7 = __.
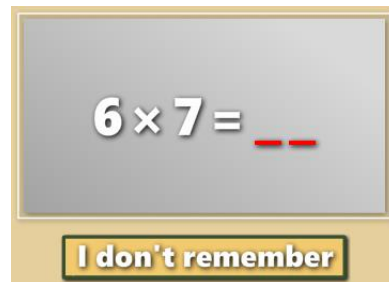
The objective of this activity is to support students in transitioning between two preliminary stages in fluency development. In the first stage, students are typically making explicit, effortful use of a strategy or relationship to derive unknown facts from known facts. As they progress through the open sentence activity, they are transitioning to retrieval conditions closer to the ultimate goal of automaticity.

Note that there is no time requirement as yet. It is important to delay the introduction of time pressure until the student is successfully recalling the fact. Studies have shown that when speed-based practice occurs before the student has made the switch to a memory-based process, it will not have the desired results. For example, Hasselbring, Goin & Bransford (1987) report on experiments in which students failed to spontaneously progress from effortful counting strategies even after as many as 70 computer-based practice sessions. Conversely, the researchers found that automaticity with the addition facts being studied could be attained in as few as 20 sessions provided that students had first successfully made the transition to retrieval of these facts from memory.

Working with open sentences in various formats also helps to develop students' competencies in several of the Common Core's mathematics standards for Operations & Algebraic Thinking:

Grade 1

1.OA.3. Apply properties of operations as strategies to add and subtract.
Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.

1.OA.4. Understand subtraction as an unknown-addend problem.
For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

Grade 2

2.OA.2. Fluently add and subtract within 20 using mental strategies.

By end of Grade 2, know from memory all sums of two one-digit numbers.

Grade 3

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers
For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

3.OA.5 Apply properties of operations as strategies to multiply and divide
If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known.

3.OA.6 Understand division as an unknown-factor problem
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division.

By the end of Grade 3, know from memory all products of two one-digit numbers.

Common Core Standards for Mathematics (2010)

C. Practice with moderate time pressure and increasing retrieval difficulty

Once the student has successfully passed through the open sentences activity, Reflex transitions to Coach Penny's Picture Puzzle. Here, the student answers facts as they appear one-by-one on tiles that cover a picture. Each correctly-answered fact removes a tile, causing the underlying picture to be progressively revealed. (The pictures are related to the game the student has chosen to play, and briefly animate after the student has answered all the facts.)

Coach Penny's Picture Puzzle marks the stage where speed of retrieval is first introduced as a performance requirement. During this stage, Reflex continuously adjusts the difficulty of the retrieval process based on the student's responses. For example, the interval between administrations of a given fact may increase or decrease, and the types of facts that are used in between administrations of that fact may be changed.

These adjustments enable Reflex to keep the difficulty of the retrieval just within the limit of the student's current ability. This maximizes the efficiency with which the strength of association of the fact in the student's memory increases with each practice trial.

The student may still occasionally have difficulty in retrieving the answer at this stage, so corrective feedback is given when necessary. Reference to the strategy used for initially acquiring the fact may also be made as appropriate. If significant difficulties persist, the fact is generally returned to explicit instruction in a subsequent session. A different strategy may be used at that point, depending on the student's performance data on the target fact and related facts.

Once the student's response patterns have stabilized and he/she is capable of consistently retrieving the fact from memory within a modest time threshold, the fact becomes "Game Ready". This means it can appear in Reflex's games.

Only facts with which the student has achieved this minimum level of recall are used in the games. This keeps gameplay fluid, and also enables the practice methods used in the games to be tailored specifically to the development of automaticity with a large set of facts at varying stages of fluency, rather than to the initial recall of a smaller set of unknown facts.

D. Game-based practice

After completing Coach Penny's Picture Puzzle, the student spends the remainder of each session playing Reflex's games. Characteristics common to all Reflex games include:

- Attractive and engaging graphics, sound effects and music.

- "Casual game" format -- simple rules and short rounds typically lasting a few minutes.

- Progressive increase in difficulty and speed of play.
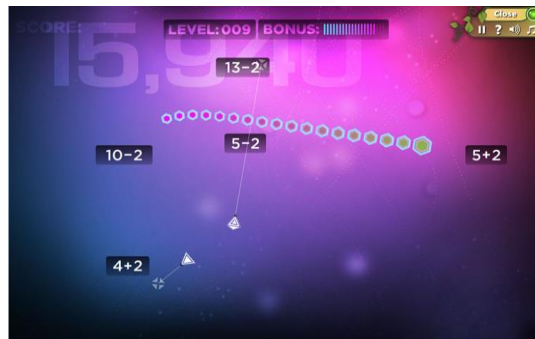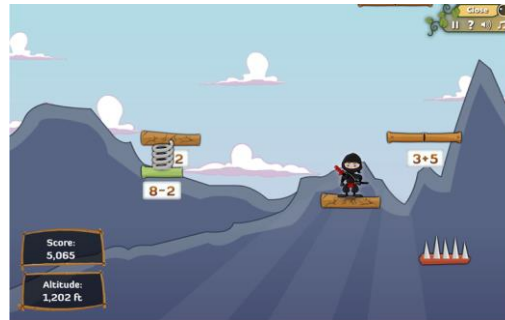
The new 'game ready' facts that the student has been studying in the current session receive extensive practice within the games. There will also typically be a larger set of additional facts that the student is working with in these games, which are at varying stages of mastery. As in Coach Penny's Picture Puzzle, facts are systematically sequenced to keep the level of challenge at the right level for that student, based on their measured retrieval speed and response patterns.

Automatization of recall via practice under increasing working memory load

As the final stage of fluency development, Reflex's games provide an environment where students must continually retrieve facts from memory at a high rate while accomplishing various objectives in the game. The key progression represented by this stage is that students are now developing the ability to retrieve facts while simultaneously dealing with significant demands on their working memory. In other words, playing a Reflex game – navigating around a maze, avoiding enemies, and achieving various game objectives -- requires the student to devote significant attention to those challenges. As players progress in a given game, its difficulty and speed will increase, placing a greater and greater load on their attention and working memory.

For example, in the game Kirie, the student frees caged animals that have been trapped by poachers. To clear each level, the student must collect a set of "keys" while avoiding enemies. Movement is controlled by answering math facts, and so the more fluidly the student can answer facts that appear in the desired direction of travel, the more adeptly he or she will be able to play the game.

As the student advances through the games, the strategic decision making required becomes more complex and fast-paced. This places an increasing load on the student's working memory. To be successful, the student must be able to quickly and effortlessly recall math facts while simultaneously scanning the game environment, paying attention to enemies' positions and movements, and so forth.

In this way, Reflex games simulate the demands that the classroom and homework will later place on the student, such as when he or she is learning new mathematical concepts and procedures. For example, when learning a process such as adding fractions with unlike denominators, Reflex students' well-honed ability to automatically retrieve math facts will enable them to be successful while their working memory is devoted to the new procedure they are mastering.

Once students have developed the ability to fluently answer math facts while achieving game objectives, you can be confident that they will be able to do the same while learning new math concepts and procedures in your classroom.

2. Adapting instruction and practice for optimal results

A key component of Reflex's effectiveness stems from its ability to take research-validated techniques for fluency development and customize both the **content** and **method** of these techniques for each student. The system continuously monitors students' response patterns for each fact under study and uses this data to differentiate instruction and adapt practice throughout a Reflex session.

> Continuously adapts
> instruction & practice to
> each student's needs
> throughout the session

The ability to customize and optimize the learning experience for individual students is a powerful feature of Reflex. Communicating this aspect of the system can be quite challenging, however. Many computer-based learning systems describe their instructional approaches as "differentiated", "individualized", and "adaptive". These terms may mean many different things, of course. Their vagueness can make it difficult to compare systems on this basis, and yet it will be increasingly critical that educators be able to make such comparisons when evaluating innovative educational technology systems that include individualization of the learning environment as a central aspect of their design.

In this section, we'll look at some specific examples of how Reflex continuously differentiates instruction and adapts practice as the student progresses through a session.

Accurate measurement of fact retrieval speed by factoring out typing ability

A relatively fundamental but still crucial aspect of Reflex's personalized learning environment is that it factors out a student's reaction speed and typing ability when measuring their current response speed with a given fact. This enables the system to make more finely-tuned measurements compared to systems that do not take this into account, particularly in the case of young children, who may be slow and/or inconsistent typists, and who generally have slower overall reactions to visual stimuli. The system is even sensitive to whether students are using the numeric keys at the top of a keyboard or the numeric keypad, since students may move between different computers – for example, using a laptop at school and a desktop computer at home when they log in over the weekend to play a couple of rounds of their favorite Reflex game.

By factoring out typing and reaction speed, Reflex is able to make highly precise and accurate measurements of fact retrieval speed. Several measurements are then combined to establish a response pattern for a given fact. This enables better and more reliable decisions to be made about the appropriate strategy or practice model to use at a given point in the student's fluency development with that fact. Systems that cannot account for variability in typing ability will be unable to accurately establish when a student has reached a high level of automaticity, let alone distinguish between different types of response patterns to the detail necessary to individualize instruction and practice on that basis.

Selecting new facts to learn, and the optimal strategy to use to learn them

Each session of Reflex begins with a brief assessment called the "Speed Cube Challenge". It serves as a progress check that enables the system to measure the student's retention of facts that were previously under study before any instruction or practice occurs in the current session. This data is combined with the longitudinal response patterns exhibited by the student over prior sessions as well those that will emerge during the current session. All this information is then used for pedagogical decision making throughout Reflex's exercises and games.

At the start of each Coaching phase, for example, fluency data from Speed Cube Challenge is combined with the progress shown by the student in previous sessions to determine the instructional focus for that session. This analysis includes both the new facts that the student has been explicitly studying, as well as those that have received significant prior practice. The system then prioritizes these facts for practice in the current session based on the student's demonstrated progress. Facts that the student may be having difficulty with will also receive different types of practice than those that are progressing smoothly to automaticity. Analysis of progress across all facts enables Reflex to make effective and efficient decisions about the overall objectives for the current session.

Another key decision that an efficient fluency development system must make when teaching new facts is selecting **which** facts to teach. This is particularly important during students' initial usage of Reflex. Choosing sets of facts that a student will have a high probability of rapid success with provides a motivational boost and builds the student's confidence. Early success in using strategies for acquisition of new facts develops students' abilities to use those strategies to master facts in the future. It also expands the pool of facts that the student has some fluency with, which increases the system's capacity to leverage existing fluency into new.

Choosing the right strategy for acquisition is also critical. The right strategy for a given student depends on several factors, including the student's degree of fluency with the fact at that point in time as well as fluency with associated facts. In this way, Reflex is customizing the method not just to the student, but to the student's current ability, as it changes over time.

Matching the pace of learning to the student's demonstrated abilities

Differentiating instruction for individual learners also requires establishing the student's readiness for new knowledge. Moving on to learning new facts before a student is ready to do so can make him or her feel overwhelmed, and also impede further progress in facts that the student has begun to master.

Reflex actively monitors each student's progress on all facts currently under study, including those from prior sessions. It will delay the introduction of new facts and focus on those covered in prior sessions in circumstances that indicate this is the best course of action for that student at that point in time.

Optimizing practice

One of the most critical aspects in the efficient development of automaticity is the difficulty of each individual retrieval during practice (Storm, Bjork, & Storm, 2010). To maximize efficiency, a fluency development system must continuously adjust the difficulty of each retrieval to be just within the student's current ability. This enables the student to maximize the "strength" of the fact in his or her memory in the shortest number of practice trials. Due to the large number of facts with which a typical student needs to develop on increase fluency, even small differences in the efficiency of practice can have significant impact on overall time to mastery of the facts in a given operation.

Reflex can increase or decrease the difficulty of retrieval by varying the interval between administrations of facts under study, as well as by changing the types of facts that appear between administrations of the study facts. By increasing the potential for interference between facts, the difficulty of the retrieval process can be increased. It is important to note that this is a key aspect of automaticity development – reliable, automatic retrieval requires a single "peak" in memory when seeing a problem such as "4 x 8", which means that the student's brain clearly associates a single response of "32" to it. Students may initially have several erroneous "peaks" (corresponding to incorrect answers) when a fact is first being acquired in long-term memory, which means that they are susceptible to occasional errors when the peaks in memory "compete" and the wrong answer is retrieved. The lack of a clear single peak may lead students to experience a feeling of limited confidence in the answer even when it is correct. Efficient practice develops increasing resistance to incorrect answers and helps the student develop rapid and reliable retrieval.

Assessing fluency

Once the student has demonstrated consistent, fluent responses with a fact that he or she has been studying, Reflex begins a 'certification' process that occurs over several days. This process examines the student's longitudinal performance with the fact under specific conditions, including the ability to successfully retrieve the fact from memory after extended periods where no practice was engaged in with that fact. In order for Reflex to certify a fact as fluent through this process, the student will need to have developed a high strength of association in memory for that fact. This kind of mastery will be robust, meaning that the student's fluency with the fact is expected to be reliably maintained over time.

Reflex will continue to periodically administer these certified facts in exercises and games in order to monitor them. Should response patterns indicate a loss of automaticity, the system has several corrective actions it can take, ranging from increasingly focused game-based practice up to a return to explicit instruction and study of the fact.

This monitoring capability is particularly valuable when an extended period of time has passed since the last usage of Reflex, such as after summer vacation. Students can re-enter Reflex for a brief refresher at the start of the school year. The system will quickly identify any facts that are exhibiting signs of reduced fluency and then support students in efficiently regaining it.

3. Recognizing effort and accomplishments

The National Math Advisory Panel has stressed the important role children's attribution of success can play in both their performance in the classroom as well as their attitude towards mathematics in general. Unfortunately, studies have shown that many children believe that success in math is dependent on an innate talent or ability, rather than being determined by the degree of effort invested in learning it – the kind of belief espoused by statements such as "I'm just not good at math".

Fortunately, research has also shown that a student's attribution of success can be changed by their experiences with mathematics in the classroom.

Children's goals and beliefs about learning are related to their mathematics performance.

Experimental studies have demonstrated that changing children's beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes: When children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning. Related research demonstrates that the engagement and sense of efficacy of African-American and Hispanic students in mathematical learning contexts not only tends to be lower than that of white and Asian students but also that it can be significantly increased.

Teachers and other educational leaders should consistently help students and parents to understand that an increased emphasis on the importance of effort is related to improved mathematics performance. This is a critical point because much of the public's self-evident resignation about mathematics education (together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the erroneous idea that success is largely a matter of inherent talent or ability, not effort.

Final Report: Main Findings and Recommendations
National Math Advisory Panel (2008), p. xxi

Math fact fluency can play a pivotal role in the development of students' attribution of success in mathematics. In a negative sense, it can reinforce the "success is due to innate talent" mindset. Many students have significant deficiencies in their fact fluency, and this can be a very public and embarrassing deficiency – students may make comments to each other such as "you don't know that??", and observe that others seem to recall facts with ease while racing through new procedures while they themselves have trouble getting past the first step. It is also understandable how a student with limited fluency might further adopt the "success comes from talent" mindset if he or she does invest significant effort in an inefficient method for developing math fact fluency, only to fail to realize noticeable gains in ability – the "labor in vain" effect noted in the literature (Nelson & Leonesio, 1988).

Yet, the challenge that the fact fluency gap poses to mathematics educators also represents a valuable opportunity to positively demonstrate how effort **can** lead to success in mathematics. Fact fluency is one of the few domains where measurable improvement can occur in almost every session, provided an effective, efficient system is employed. These steady gains can be an incredibly powerful motivator for the student, instilling confidence that goes beyond the realm of math facts and builds perseverance for tackling new challenges in all subjects (e.g., Dweck, 2006).

Reflex has been specifically designed to support this important message in several ways.

Individualized progression through each stage of fluency development ensures that students are adequately challenged and engaged, but also kept within their current level of proficiency.

Visible, continuous growth in fluency starts early in a student's usage of Reflex. Most students will make significant fluency gains within their first two weeks of Reflex usage.

Importantly, gains will continue occurring regularly from that point on. This motivates the student to log in more frequently, even on their own from outside the classroom.

Recognition of effort as well as progress.

In addition to receiving tokens when they master facts and fact families, students also receive additional tokens based on the number of correct responses they made in a given part of the system. This rewards the effort they have invested, even in those sessions where they have not achieved full fluency in a new fact or family.

Students also see their progress through the progressive filling in of their "Family Pyramid" and fact tables for individual operations:

The Family Pyramid (top to bottom):

- 144 / 12 12
- 121 / 11 11 — 132 / 11 12
- 100 / 10 10 — 110 / 10 11 — 120 / 10 12
- 81 / 9 9 — 90 / 9 10 — 99 / 9 11 — 108 / 9 12
- 64 / 8 8 — 72 / 8 9 — 80 / 8 10 — 88 / 8 11 — 96 / 8 12
- 49 / 7 7 — 56 / 7 8 — 63 / 7 9 — 70 / 7 10 — 77 / 7 11 — 84 / 7 12
- 36 / 6 6 — 42 / 6 7 — 48 / 6 8 — 54 / 6 9 — 60 / 6 10 — 66 / 6 11 — 72 / 6 12
- 25 / 5 5 — 30 / 5 6 — 35 / 5 7 — 40 / 5 8 — 45 / 5 9 — 50 / 5 10 — 55 / 5 11 — 60 / 5 12
- 16 / 4 4 — 20 / 4 5 — 24 / 4 6 — 28 / 4 7 — 32 / 4 8 — 36 / 4 9 — 40 / 4 10 — 44 / 4 11 — 48 / 4 12
- 9 / 3 3 — 12 / 3 4 — 15 / 3 5 — 18 / 3 6 — 21 / 3 7 — 24 / 3 8 — 27 / 3 9 — 30 / 3 10 — 33 / 3 11 — 36 / 3 12
- 4 / 2 2 — 6 / 2 3 — 8 / 2 4 — 10 / 2 5 — 12 / 2 6 — 14 / 2 7 — 16 / 2 8 — 18 / 2 9 — 20 / 2 10 — 22 / 2 11 — 24 / 2 12
- 1 / 1 1 — 2 / 1 2 — 3 / 1 3 — 4 / 1 4 — 5 / 1 5 — 6 / 1 6 — 7 / 1 7 — 8 / 1 8 — 9 / 1 9 — 10 / 1 10 — 11 / 1 11 — 12 / 1 12
- 0 / 0 0 — 0 / 0 1 — 0 / 0 2 — 0 / 0 3 — 0 / 0 4 — 0 / 0 5 — 0 / 0 6 — 0 / 0 7 — 0 / 0 8 — 0 / 0 9 — 0 / 0 10 — 0 / 0 11 — 0 / 0 12

Multiplication fact table:

0 × 0	0 × 1	0 × 2	0 × 3	0 × 4	0 × 5	0 × 6	0 × 7	0 × 8	0 × 9	0 × 10	0 × 11	0 × 12
1 × 0	1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 0	2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 0	3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 0	4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 0	5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 0	6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 0	7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 0	8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 0	9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 0	10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 0	11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 0	12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

Division fact table:

0 ÷ 1	0 ÷ 2	0 ÷ 3	0 ÷ 4	0 ÷ 5	0 ÷ 6	0 ÷ 7	0 ÷ 8	0 ÷ 9	0 ÷ 10	0 ÷ 11	0 ÷ 12
1 ÷ 1	2 ÷ 2	3 ÷ 3	4 ÷ 4	5 ÷ 5	6 ÷ 6	7 ÷ 7	8 ÷ 8	9 ÷ 9	10 ÷ 10	11 ÷ 11	12 ÷ 12
2 ÷ 1	4 ÷ 2	6 ÷ 3	8 ÷ 4	10 ÷ 5	12 ÷ 6	14 ÷ 7	16 ÷ 8	18 ÷ 9	20 ÷ 10	22 ÷ 11	24 ÷ 12
3 ÷ 1	6 ÷ 2	9 ÷ 3	12 ÷ 4	15 ÷ 5	18 ÷ 6	21 ÷ 7	24 ÷ 8	27 ÷ 9	30 ÷ 10	33 ÷ 11	36 ÷ 12
4 ÷ 1	8 ÷ 2	12 ÷ 3	16 ÷ 4	20 ÷ 5	24 ÷ 6	28 ÷ 7	32 ÷ 8	36 ÷ 9	40 ÷ 10	44 ÷ 11	48 ÷ 12
5 ÷ 1	10 ÷ 2	15 ÷ 3	20 ÷ 4	25 ÷ 5	30 ÷ 6	35 ÷ 7	40 ÷ 8	45 ÷ 9	50 ÷ 10	55 ÷ 11	60 ÷ 12
6 ÷ 1	12 ÷ 2	18 ÷ 3	24 ÷ 4	30 ÷ 5	36 ÷ 6	42 ÷ 7	48 ÷ 8	54 ÷ 9	60 ÷ 10	66 ÷ 11	72 ÷ 12
7 ÷ 1	14 ÷ 2	21 ÷ 3	28 ÷ 4	35 ÷ 5	42 ÷ 6	49 ÷ 7	56 ÷ 8	63 ÷ 9	70 ÷ 10	77 ÷ 11	84 ÷ 12
8 ÷ 1	16 ÷ 2	24 ÷ 3	32 ÷ 4	40 ÷ 5	48 ÷ 6	56 ÷ 7	64 ÷ 8	72 ÷ 9	80 ÷ 10	88 ÷ 11	96 ÷ 12
9 ÷ 1	18 ÷ 2	27 ÷ 3	36 ÷ 4	45 ÷ 5	54 ÷ 6	63 ÷ 7	72 ÷ 8	81 ÷ 9	90 ÷ 10	99 ÷ 11	108 ÷ 12
10 ÷ 1	20 ÷ 2	30 ÷ 3	40 ÷ 4	50 ÷ 5	60 ÷ 6	70 ÷ 7	80 ÷ 8	90 ÷ 9	100 ÷ 10	110 ÷ 11	120 ÷ 12
11 ÷ 1	22 ÷ 2	33 ÷ 3	44 ÷ 4	55 ÷ 5	66 ÷ 6	77 ÷ 7	88 ÷ 8	99 ÷ 9	110 ÷ 10	121 ÷ 11	132 ÷ 12
12 ÷ 1	24 ÷ 2	36 ÷ 3	48 ÷ 4	60 ÷ 5	72 ÷ 6	84 ÷ 7	96 ÷ 8	108 ÷ 9	120 ÷ 10	132 ÷ 11	144 ÷ 12

Games

Well-designed games provide a practice environment that students find enjoyable and engaging. This can be particularly important for students that struggle to maintain focus with traditional forms of practice -- for example, computer games can result in more sustained attention and reduced impulsivity for students with ADHD, compared to standard computerized tasks (Ford, Poe & Cox, 1993; Shaw Grayson & Lewis, 2005).
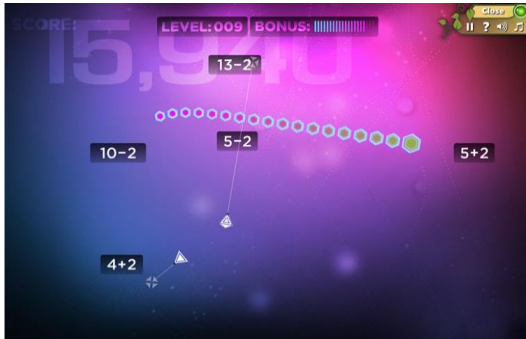
Students' enjoyment of Reflex's games often leads them to keep playing well beyond the daily usage requirement when classroom time allows, and to log in for additional game sessions on their own time outside school.

Additional fun

Once a student has received sufficient fluency practice in a given session, the Reflex Store "unlocks" and students can enter it to spend the tokens that have been earned from progress and effort. The Store contains a wide selection of fun items to use to customize the student's "avatar".

The most exotic and elaborate accessories require the most tokens, motivating students to continue frequent use of Reflex and perform to the best of their abilities during each session in order to maximize the number of tokens they receive.

4. Monitoring progress

In addition to being a powerful solution for developing students' fact mastery, Reflex also provides educators with intuitive reporting and administrative features that enable them to easily monitor progress and effectively manage their students' usage of the Reflex system.

This can be a particularly important capability when Reflex is used within an intervention context. For example, addressing fact fluency deficiencies is a common part of interventions conducted within the Response To Intervention (RTI) framework. In addition to advocating daily fluency practice, the What Works Clearinghouse's RTI guidelines for mathematics also recommend frequent progress monitoring to evaluate how well individual students are responding to the intervention (Gersten et al., 2009).

Easy-to-use reporting
to monitor students'
usage & progress

Reflex's reporting system enables educators to stay on top of their fluency intervention, including:

- "Dashboard"-style overviews of key information on usage and progress, as well as alerts when key individual and group milestones are reached or if usage drops below recommended minimum levels.

- Summary and detailed data on current fluency levels and daily fluency growth for both individuals and groups of students, with the ability to separate pre-existing fluency from new fluency gained within Reflex.

- Displays that relate Reflex usage to fluency gains. This also supports the identification of students not responding to a fluency-focused intervention that involves Reflex; for example, due to insufficient usage or lack of readiness, such as an inability to count.
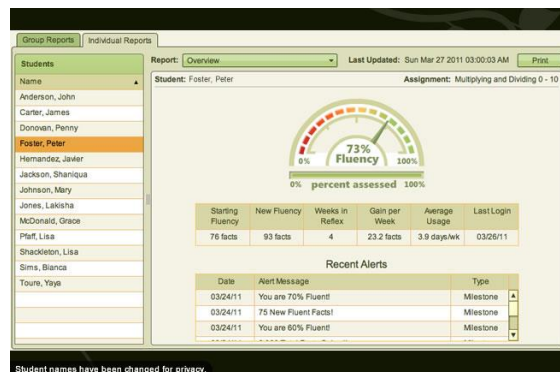
Summary

Extensive research has shown that students who are not fluent with their basic math facts have major challenges mastering more advanced mathematical skills. The NCTM, National Math Advisory Panel, and other major educational organizations have all recently highlighted math fact fluency as a central pillar of every student's mathematics education.

The Reflex system combines research-proven methods with innovative technology to enable students of all ability levels to master their math facts.

Adaptivity and individualization
Reflex continuously monitors each student's performance to create the optimal experience for every child.

Intuitive and powerful reporting
Educators have everything they need to easily monitor and support student progress.

Anytime, anywhere access
Students can build fluency with Reflex anywhere there is an Internet connection.
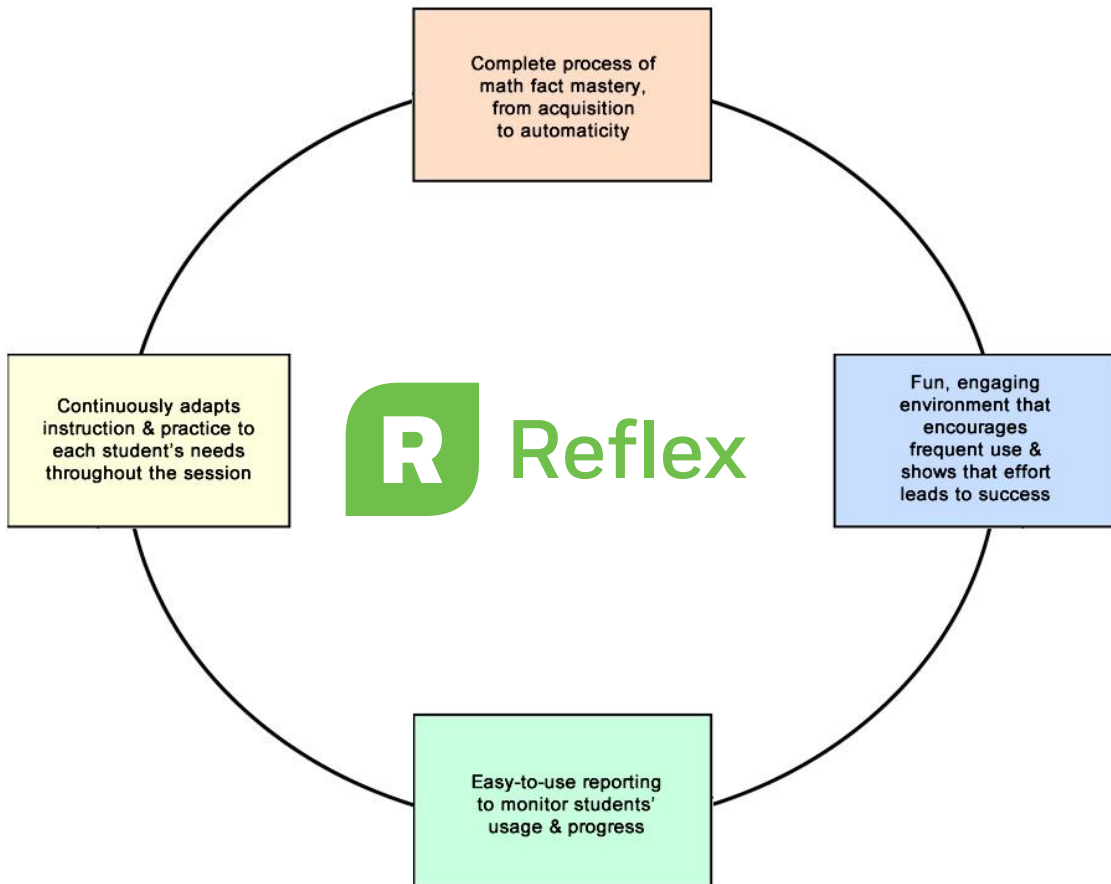
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